MATHEMATICS LEARNING THROUGH HISTORY OF MATHEMATICS.

FROM ARAB MATHEMATICIAN OMAR-AL-KHAYYAM METHOD.

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ABSTRACT :

This paper illustrates the significance of the geometrical representation, Arabic culture and modern technology Cabri geometry. In this regards I posed history of mathematics. In this case I used Omar-Al-khayyam's method wherein he used to solve cubic equation considering unknown numbers. I used known numbers and justify modern technology and change student's idea about mathematics.

1. Introduction,.
2. The purpose of study and the aim of study,
3. An contents of class,
   3.1 Explanation of materials,
   3.2 Surroundings of the class,
   3.3 Development of class,
4. Result of questionnaire.
5. Conclusion.

Key Word: Solid, cubic equation, parabola, Hyperbola , square.

1. INTRODUCTION :-

Mathematics is a subject which student doesn't feel interest to solve. The teacher who knows little of the history of Mathematics is apt to teach techniques in isolation, unrelated to the problems and ideas which generated them. History exposes interrelations among different mathematical domains, or, of mathematics with other discipline (for example culture). History tells us that the tools used for mathematical inquiry are themselves reformulated using mathematical (especially geometrical) representations, as mathematicians in the scientific revolution tried to select, find or construct convenient representations and instruments for their research. This paper also discussed software Cabri Geometry from the view point of integrating traditional instruments (compass, ruler etc.) to Modern Technology.

Historically we can find that Arab mathematician Omar-al-Khayyam, Giyat ed-din Abul Fath, was born at Nishabur(IRAN) 1048 A.D. and died 1131 A.D. He could solve algebraic cubic equation by using geometrical representation. For solving three degree (cubic) equation he used Greek mathematical work, Euclid and Apollonius. Omar made no addition to the theory of conics, he did apply the principle of intersecting conic sections to solving algebraic problems. In doing so he not only exhibited his mastery of conic sections, but also showed that he was aware of the practical applications of what was a highly abstruse area of geometry.

History of Arab Mathematics :-

The year A.D. 622 is a momentous one in World history. The Prophet Muhammad fled from Mecca to Medina, about 350 kilometres away and spread the wholly Quran and established Islam. There is a long history, I would like to overtake that, cause of mathematics history of Arab especially Baghdad (IRAN). The process was aided by the creative tension between two main traditions of astronomy and mathematics represented in Baghdad, even from the early years of Islamic rule. One tradition derived directly from India and Persian sources and is best exemplified in the astronomical tables and the algebraic approach to mathematics. One of the greatest exponents of this tradition, who left an indelible mark on the subsequent development of Arab mathematics, was al-Khwarizmi. To him mathematics had to be useful and help with practical corners such as determining inheritances, constructing Calenders or religious observances. The other tradition looked to Hellenistic mathematics, with its strong emphasis on geometry and deductive methods. They established a school in Baghdad. One of the best-known exponents of this school was Thabit ibn Qurra, who was both an outstanding translator of Greek texts and an original contributor to geometry and algebra. That the two traditions eventually merged is evident in the work of later Arab mathematicians such as Omar Khayyam and al-Kashi. Abu Jafar Muhammad ibn Musa-al-Khwarizmi (780-850AD) wrote Hisab al-jabr w'al-muqabala (calculation by restoration and reduction). His book on algebra contained an analysis of property relations and the distribution of inheritance according to Islamic law, and rules for drawing up walls. But he technically avoid three degree equation. In this tradition the other scientists who were largely responsible for lying the foundations of modern science. These included Thabit ibn Qurra (836-901), al-Razi (865-901), al-Haytham 8965-10399,al-Biruni (973-1051), ibn Sina (980-1037), Omar al-Khayyam (1048-1131), Sharaf al-din al-Tusi (d1213), Nasir al-Din al Tusi (1201-74), and al-Kashi (d 1429). Among them Ibrahim ibn Sina's commentary extending Archimedes work on the quadrature of the parabola has been described as one of the most innovative approaches known before the emergence of the integral calculus.

2. THE PURPOSE OF STUDY AND THE WAY OF STUDY :-

The purpose of study : - I would like to change clearly students idea through experiencing by history of mathematics.

The way of study : - To achieve the above mentioned aim I established the following :-

i The students may be don't know the meaning of geometrical representation of algebraic equation. Through my lecture they'll learn that.

ii May be they don't have any idea about Arab culture, though my lecture they will introduce a new culture.

iii Through my lecture they could apply old mathematics in Modern Technology.

For the above purposes I posed questionnaire for students before the lecture and after the lecture.

3. An outline of the class.

3.1 Explanation of materials : -

The Algebra of Omar Khayyam (page 75 to 79), Euclid Elements (pages 69-70),Euclid Archimedes Apollonius of perga (pages 616-618, 627-628,640,669-670,661-662), Rubaiyat of Omar Khayyam (original Persian language pages 3-6, poem 17) Arab Genius in science and Philosophy (page 48, original Arabic language).

3.2 Surroundings of the class : -

(1)The object of study : -

Ten undergraduate students, six of them are fourth grade, two of them are third grade, two of them are second grade undergraduate students. Among them four students are related mathematics and six students are related human science.

(2)**Preparation**

Computer (Windows), Cabri Geometry II, Microsoft Power Point, Video Projector, before lecture questionnaire, after lecture questionnaire, Text book, Work sheet, Home work sheet.

3.3 **The development of the lesson** : -

For studying Omar khayyam as an ancient great mathematician the text-book based on Historical type was developed. He was the first who did solve algebraic cubic equations by using geometrical representation. To solve cubic equation he took help from Greek mathematics. He applied Euclid famous "Two mean proportion" formula and Apollonius conic section for solving cubic equations. He solved cubic trinomial and tetranomial equations. He always considered a solid (rectangular parallelogram).

First lesson : - **Problem1**:- A cube and a sides are equal a number i.e. $x^3+bx=a$.

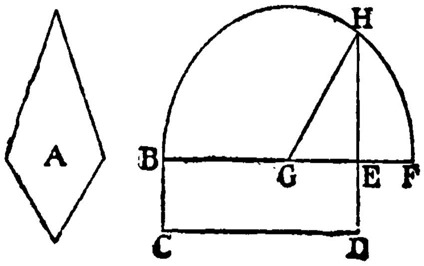
Before solving the problem I confirmed students knowledge with using historical text such as Euclidean Elements, definition of Apollonius parabola.

i .Two mean proportion : -

To describe a square that shall be equal to a given rectilinear figure, whatever known as two mean proportion.

Description :- Let A be a rectilinear figure : it is required to describe a square that shall be equal to A.

How :- The rectangular parallelogram BCDE equal to the rectilinear figure A. If BE = ED, then it is a square. Produce BE to F such that EF = ED. On BF describe a semi-circle, center at G, produce DE to H. Join GH. Then, because the straight line BF is divided into two equal parts at the point G, and into two unequal parts at the point E, the rectangle BE, EF, together with the square on GE, is equal to the square on GF. But DE = EF. So square on EH is equal to BCDE. If EH = x, BE = a and ED = EF = b, then $x^2 = a \circ b$.



This Euclidean theorem is known as two mean proportion.

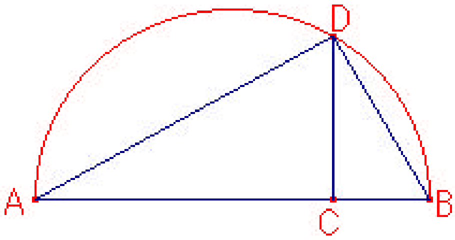
Mathematically :-

AC=a

BC=b

DC=x

$X^2 = a \cdot b$



ii. Definition of a Parabola : -

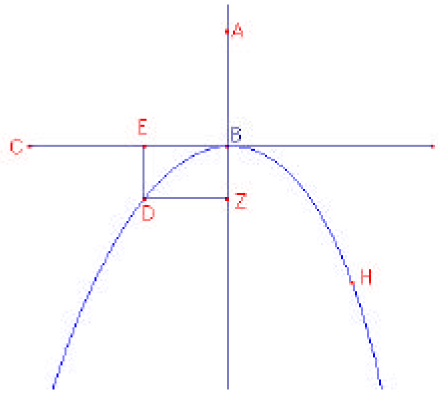
In this case I used definition of parabola from Apollonius, whatever historical.

AB = parameter

BZ = diameter

DZ = ordinate

$DZ^2 = AB \cdot BZ$



iii. Solid (rectangular parallelogram) : -

In this case I used a model of solid.

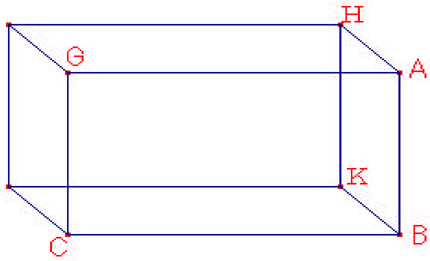
In the figure ABCG is a solid, whose side AB i.e. ABKH is a square. AB is the given number of roots b, so $AB^2 = b$.

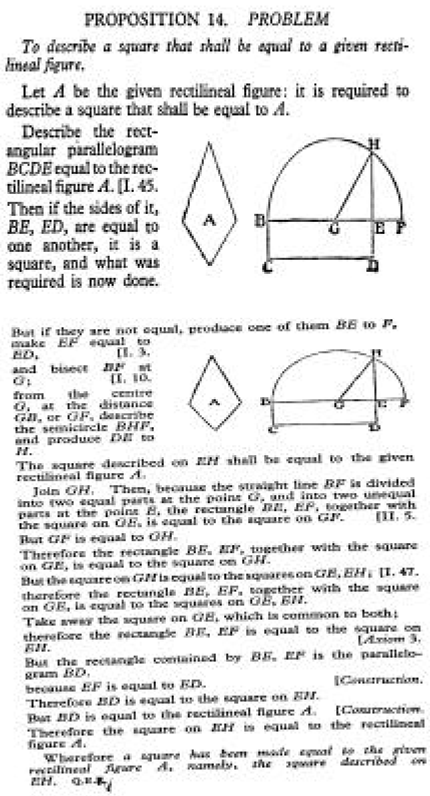
Volume of the solid is a

i.e. $AB^2.BC=a$

Height $BC=a/b$

BC is perpendicular to AB.





PROPOSITION 14. *PROBLEM*

To describe a square that shall be equal to a given rectilineal figure.

Let *A* be the given rectilineal figure: it is required to describe a square that shall be equal to *A*.

Describe the rectangular parallelogram *BCDE* equal to the rectilineal figure *A*. [I. 45.

Then if the sides of it, *BE*, *ED*, are equal to one another, it is a square, and what was required is now done.

But if they are not equal, produce one of them *BE* to *F*, make *EF* equal to *ED*, [I. 3.

and bisect *BF* at *G*; [I. 10.

from the centre *G*, at the distance *GB*, or *GF*, describe the semicircle *BHF*, and produce *DE* to *H*.

The square described on *EH* shall be equal to the given rectilineal figure *A*.

Join *GH*. Then, because the straight line *BF* is divided into two equal parts at the point *G*, and into two unequal parts at the point *E*, the rectangle *BE*, *EF*, together with the square on *GE*, is equal to the square on *GF*. [II. 5.

But *GF* is equal to *GH*.

Therefore the rectangle *BE*, *EF*, together with the square on *GE*, is equal to the square on *GH*.

But the square on *GH* is equal to the squares on *GE*, *EH*; [I. 47.

therefore the rectangle *BE*, *EF*, together with the square on *GE*, is equal to the squares on *GE*, *EH*.

Take away the square on *GE*, which is common to both; therefore the rectangle *BE*, *EF* is equal to the square on *EH*. [Axiom 3.

But the rectangle contained by *BE*, *EF* is the parallelogram *BD*, because *EF* is equal to *ED*. [Construction.

Therefore *BD* is equal to the square on *EH*.

But *BD* is equal to the rectilineal figure *A*. [Construction.

Therefore the square on *EH* is equal to the rectilineal figure *A*.

Wherefore a square has been made equal to the given rectilineal figure A, namely, the square described on EH. Q.E.F.

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与えられた直線図形に等しい正方形をつくること。

与えられた直線図形を *A* とせよ。このとき直線図形 *A* に等しい正方形をつくらねばならぬ。

直線図形 *A* に等しい直角平行四辺形 *BΔ* がつくられたとせよ。そうすればもし *BE* が *EΔ* に等しければ，命じられたことはなされたことになるであろう。なぜなら正方形 *BΔ* が直線図形 *A* に等しくつくられたから。もし等しくなければ，*BE*，*EΔ* の一方が大きい。*BE* が大きいとし，*BE* が *Z* まで延長され，*EZ* が *EΔ* に等しくされ，*BZ* が *H* で2等分され，*H* を中心とし，*HB*，*HZ* の一を半径として半円 *BΘZ* が描かれ，*ΔE* が *Θ* まで延長され，*HΘ* が結ばれたとせよ。

そうすれば線分 *BZ* は *H* において等しい部分に，*E* において不等な部分に分けられたから，*BE*，*EZ* にかこまれた矩形と *EH* 上の正方形との和は *HZ* 上の正方形に等しい。そして *HZ* は *HΘ* に等しい。それゆえ矩形 *BE*，*EZ* と *HE* 上の正方形との和は *HΘ* 上の正方形に等しい。ところが *ΘE*，*EH* 上の正方形の和は *HΘ* 上の正方形に等しい。ゆえに矩形 *BE*，*EZ* と *HE* 上の正方形との和は *ΘE*，*EH* 上の正方形の和に等しい。双方から *HE* 上の正方形がひかれたとせよ。そうすれば残りの *BE*，*EZ* にかこまれた矩形は *EΘ* 上の正方形に等しい。ところが *EZ* は *EΔ* に等しいから，矩形 *BE*，*EZ* は *BΔ* である。それゆえ平行四辺形 *BΔ* は *ΘE* 上の正方形に等しい。そして *BΔ* は直線図形 *A* に等しい。ゆえに直線図形 *A* も *EΘ* 上に描かれた正方形に等しい。

よって与えられた直線図形 *A* に等しい正方形，すなわち *EΘ* 上に描かれうる正方形がつくられた。これが作図すべきものであった[1]。

برچهره گل شبنم نوروز خوش است در طرف چمن روی دل افروز خوش است

از دی که گذشت هر چه گویی خوش نیست خوش باش و ز دی مگو که امروز خوش است

* * *

الدرجة الأولى والثانية والثالثة والرابعة. واستطاع عمر الخيام (ت ٥١٧ هـ - ١١٢٣ م) حل المعادلات من الدرجة الثالثة والرابعة بواسطة قطع المخروط، وهذا أرقى ما وصل اليه العرب في الجبر، بل من أرقى ما وصل اليه علماء الرياضيات في حل المعادلات في الوقت الحاضر، لأننا نجهل اليوم كيفية حل المعادلة من الدرجة الخامسة وما فوقها بطريقة عامة (٢)، ولقد اطرى كاجوري (ص ١٠٧) عمر الخيام ثم قال (٣): «ان حل المعادلات التكعيبية بواسطة قطوع المخروط من اعظم الاعمال التي قام بها العرب».

In English.

XX

Ah, my Belovéd, fill the cup that
clears
To-day of past Regrets and future
Fears—
To-Morrow?—Why, To-morrow I
may be
Myself with Yesterday's Sev'n Thou-
sand Years.

OMAR-Al-KHAYYAM could solve three degree equation from third and fourth term by using geometrical shape, and this is the highest peak (top) reached by ARAB'S in algebra and it is higher than what some mathematician have reached now-a-days, because still now we can not solve equations from fifth degree or higher by a common way. Gregory had praised Omar Khayyam, and said, "solving cubic equations by using geometrical representation, was one of the greatest achievements of ARAB scientists"

These above are from original book whatever I used in the lectures.

Now we posed first problem of Omar Khayyam, below.

I. The first species. *A cube and sides are equal to a number.*[1]

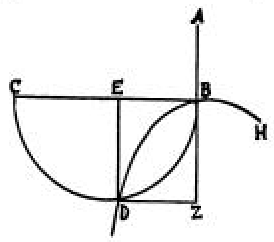
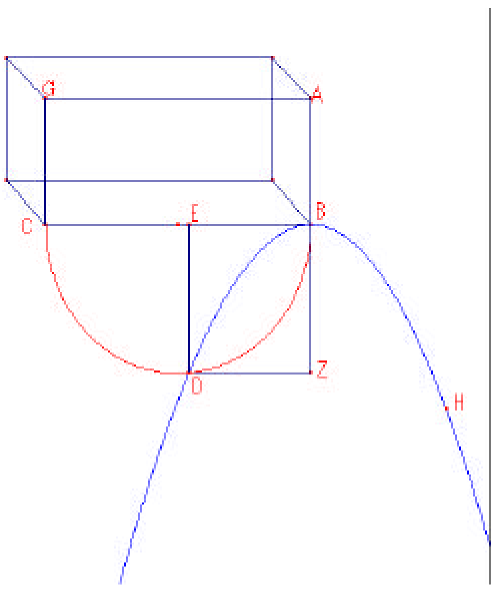


FIG. 17

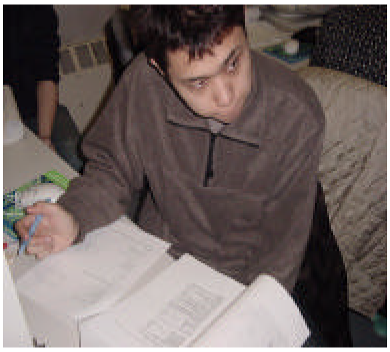
Let the line AB (Fig. 17) be the side of a square equal to the given number of roots.[2] Construct a solid whose base is equal to the square on AB, equal in volume to the given number. The construction has been shown previously.[3] Let BC be the height of the solid. Let BC be perpendicular to AB. You know already what meaning is applied in this discussion to the phrase *solid number*. It is a solid whose base is the square of unity and whose height is equal to the given number; that is, the height is a line whose ratio to the side of the base of the solid is as the ratio of the given number to one. Produce AB to Z and construct a parabola whose vertex is the point B, axis BZ, and parameter AB. Then the position of the conic HBD will be known, as has been shown previously and it will be tangent to BC. Describe on BC a semicircle. It necessarily intersects the conic. Let the point of intersection be D; drop from D, whose position is known, two perpendiculars DZ and DE on BZ and BC. Both the position and the magnitude of these lines are known. The line DZ is an ordinate of the conic. Its square is then equal to the product of BZ and AB.[5] Consequently, AB to DZ, which is equal to BE, is as BE to ED, which is equal to ZB.[6] But BE to ED is as ED to EC.[7] The four lines then are in continuous proportion, AB, BE, ED, EC,[8] and consequently the square of the parameter AB, the first, is to the square of BE, the second, as BE, the second, is to EC, the fourth.[9] The solid whose base is the square AB and whose height is EC is equal to the cube BE, because the heights of these figures are reciprocally equal to their bases.[10] Let the solid whose base is the square of AB and height is EB be added to both.[11] The cube BE plus the solid then is equal to the solid whose base is the square AB and whose height is BC, which solid we have assumed to be equal to the given number. But the solid whose base is the square of AB, which is equal to the number of roots, and whose height is EB, which is the side of the cube, is equal to the number of the given sides of the cube EB. The cube EB, then, plus the number of its given sides is equal to the given number, which was required.

This species does not present varieties of cases or impossible problems. It has been solved by means of the properties of the circle combined with those of the parabola.



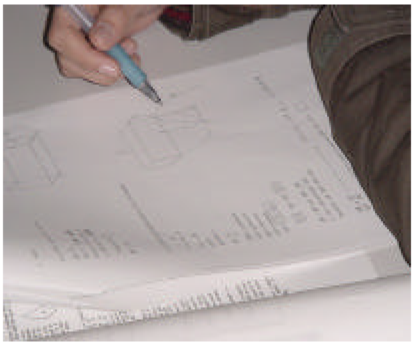
A cube and a side equal to a number.

i.e. $X^3 + bX = a$.



Student thinking.

Understanding and solving



Understanding and solving

In the first lesson I used a model of a solid and describe smoothly and asked the following questions justifying their understanding.

T : AB is the given number of roots b so AB^2 ?

S1 : b.

T : Volume of the solid ABCG is a, AB is the side is a square is b, BC is the height of the solid and perpendicular to AB so mathematically BC=?

S2 : BC = a/b.

T : DBH, a parabola whose

S3 : Vertex at A.

S4 : AB= Parameter.

S5 : BZ = Diameter.

S6 : DZ= ordinate.

S7 : So from the definition of parabola $DZ^2=AB.BZ$.

Draw a semicircle CDB so that the semi circle and the parabola necessarily meet at the point D. Now draw perpendicular DZ to ZB and DE to BC. Then BE=DZ and ED=BZ. Now BEDZ is a rectangle. I also told them to follow the worksheet and try to fill up that. They did that.

Discussion about the class : -

This cubic equation is possible only for the positive unknown numbers. If the semicircle CDB and the parabola DBH does not meet then the solution is impossible. Omar's geometrical methods were more comprehensive than the algebraic methods developed by the Italian algebraists, notably Girolano Cardano and Niccolo Tartaglia

For the next class I divide the number of students in two groups. I distribute two problems as their homework.

These two problem are as follows:-

II.THE SECOND SPECIES. A CUBE AND A NUMBER ARE EQUAL TO SIDES.

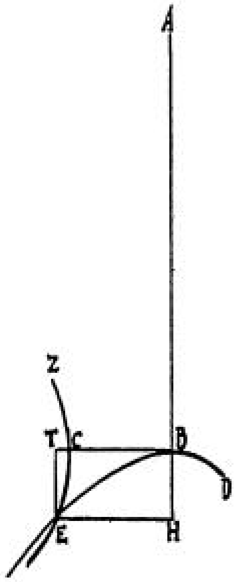


FIG. 18

Let the line AB (Fig.18) be the side of a square equal to the number of the roots, and construct a solid having as its base the square of AB and equal to the given number, and let its height BC be perpendicular to AB. Describe a parabola having as its vertex the point B and its axis along the direction AB and its parameter AB. This is, then, the curve DBE, whose position is known. Construct also a second conic, namely, a hyperbola whose vertex is the point C and whose axis is arong the direction of BC. Each one of its two parameters, the perpendicular and the oblique, is equal to BC. It is the curve ECZ. This hyperbola also is known in position, as was shown by Apollonius in the 58th proposition of his first book. The two conics will either meet or will not meet. If they do not meet, the problem is impossible of solution. If they do meet, they do it tangentially at a point or by intersection at two points.

Suppose they meet at a point and let it be at E, whose position is known. Then drop from it two perpendiculars ET and EH on the two lines BT and BH. The two perpendiculars are known unerringly in position and magnitude. The line ET is an ordinate of the hyperbola. Consequently, the square of ET is to the product of BT and TC as the parameter is to the oblique, as was demonstrated by Apollonius in the twentieth proposition of the first book. The two sides, the perpendicular. and the oblique, are equal. Then the square ET is equal to the product of BT and TC, and BT to TE is as TE to TC. But the square of EH, which is equivalent to BT, is equal to the product of BH and BA, as was demonstrated in the second proposition of the first book of the treatise on conics.Consequently, AB is to BT as BT is to BH and as BH, which is equal to ET, is to TC. The four lines, AB, BT, ET, TC, then, are in continuous proportion, and the square of AB, the first, is to the square of BT, the second, as BT, the second, is to TC, the fourth. The cube of BT, then, is equal to the solid whose base is the square AB and whose height is CT. Let the solid whose base is the square of AB and whose height is BC, which was made equal to the given number, be added to both. Then the cube BT plus the given number is equal to the solid whose base is the square of AB and whose height is BT, which represents the number of the sides of the cube.

Thus it is shown that this species includes different cases and among its problems are some that are impossible. The species has been solved by means of the properties of the two conics, the parabola and the hyperbola.

III.THE THIRD SPECIES. A CUBE IS EQUAL TO SIDES PLUS A NUMBER.

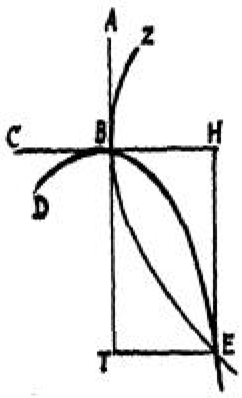


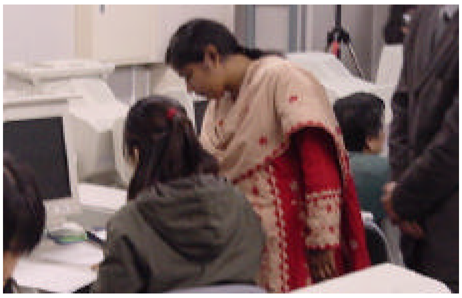
FIG. 19

Let AB (Fig.19), equivalent to the side of a square, which is equal to the number of the sides. Construct a solid whose base is the square of AB, and equal to the given number, and let its height BC be perpendicular on AB. Prolong AB and BC and describe a parabola whose vertex is the point B, whose axis is the prolongation of AB and whose parameter is AB. This is the parabola DBE, which is of known position and is tangent to the line BH as Apollonius has shown in the thirty-third proposition of the first book. Then describe another conic, a hyperbola whose vertex is the point B and whose axis is the prolongation of BC, having both of its parameters, the perpendicular and the oblique, equal to BC. This is the hyperbola ZBE, which is of known position, and tangent to the line AB. The two conics must necessarily intersect. Let them intersect at the point E whose position is known. Let fall from the point E the perpendiculars ET and EH. They are known in position and magnitude and the line EH is an ordinate of the hyperbola. As it was shown above, its square is equal to the product of CH and BH. Then CH to EH is as EH to HB, but EH, which is equal to BT, is to HB as ET is to AB, the parameter of the parabola. The four lines, then, are in continuous proportion. AB to HB is as HB to BT and as BT to CH. Then the square of AB, the first, to the square of HB, the second, is as HB, the second, to CH, the forth. The cube of HB, then, is equal to the solid whose base is the square of AB and whose height is CH, as both of their heights are reciprocally proportional to their bases. But this solid is equal to the solid whose base is the square of AB and whose height is BC. The solid was known to be equal to the given number plus the solid bounded by a base equal to the square of AB and a height BH, the later being equal to the number of sides given to the cube BH. Then cube BH is equal to the given number plus the given number of sides, which was required.

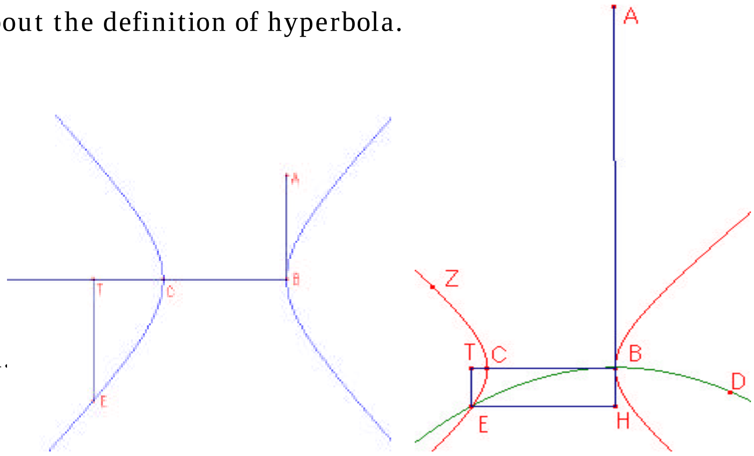
It has been shown that this species has no varieties of cases and has no impossible problems. It was solved by means of the properties of the parabola combined with those of a hyperbola.

From the beginning of the second lecture I divide the all number of students into two groups for two problems and made leader who present the problem in the blackboard. They discussed themselves.

For their understanding I made clear about the definition of hyperbola.



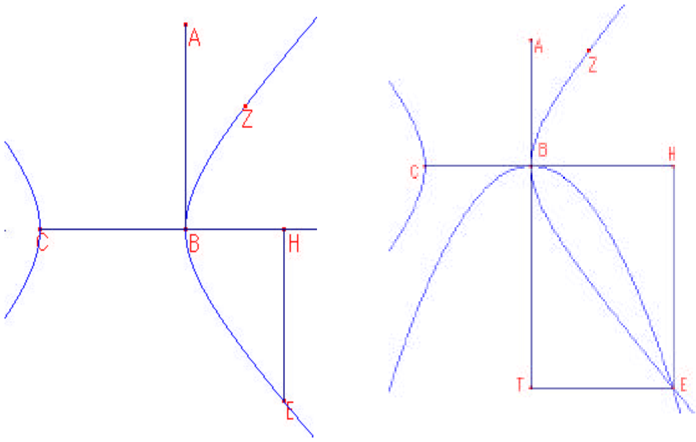
I helped her to understand the problem.



A cube and a number are equal to sides i.e.

$X^3 + a = bx$.

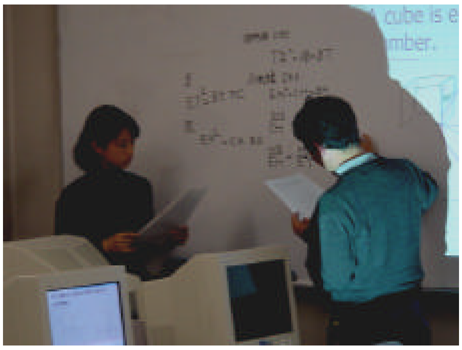
I helped to understand individually.



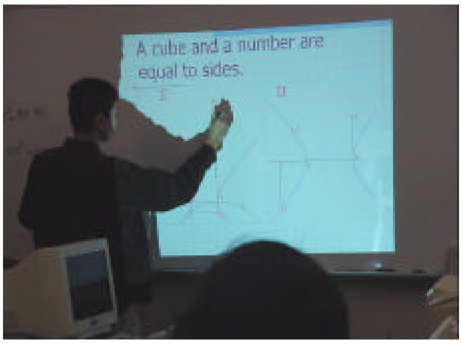
A cube is equal to sides plus a number

I.e., $X^3 = bX + a$.

A group thinking how to solve.



One group is describing.



Other group leader is describing.

For the third lecture I posed some known numbers instead of the volume of the solid a, and the side b is 6 and 2 respectively.

So from last two lecture height = volume / sides.

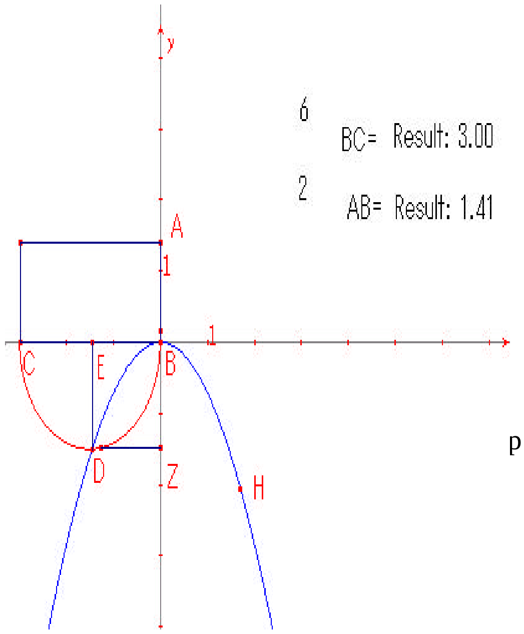
BC = 6 /2.

= 3.

And root 2 is 1.41.

Using previous formula students drew $x^3 + 2x = 6$ (original equation is $x^3 + bx = a$).

The following are the Cabri Geometry whatever students draw. We also helped them.



ped to draw figure in computer.

Student is very active in computer.

At the end of the third lecture I compared Omar and Cardano's method for cubic equation. After then we conclude that Omar is the father of cubic equation.

4. **RESULT OF THE QUESTIONNAIRE** : -

Before the lecture I posed MCQ (multiple choice question) type question. Which are below:-

Q : - Do you have any idea about Arab and their culture.

A : - Most of them write no, some of them write Arabia language.

Q : - Do you have any idea about geometric representation.

A : - No.

Q : - In general whatever you use to draw figure.

A : - Pencil, Compass, Ruler.

Since my students are University undergraduate. So after my three lectures I asked the following question and justifying their understanding.

For modern technology I posed the following question to students.

Q : - If you are teacher how can you teach Omar Khayyam's mathematics.

A : - If I'll be a teacher I use Cabri Geometry, because there is no difficulty in handling.

A : - It is possible to draw Parabola by the software of CABRI GEOMETRY. So, I'll teach students by Personal Computer.

A : - It is easy to draw figure by using Cabri Geometry. Using this Software we can also justify and confirm about our figure.

For the culture I posed the following question.

Q : - What are the time and cultural difference between Omar mathematics and the mathematics you have learned.

A : - Comparison now and long time ago, We learn mathematics whatever mathematician made、but Omar made mathematics himself.

A : - We and mathematician are different standpoint. We learn only pattern mathematics. As a result different time culture idea is different.

A : - By turning my attention to regional mathematics differ from place to place. I have learned different through the three lectures. I feel diversity (variety).

About the geometrical representation I posed the following question.

Q : - Through three lecture what you have learned new.

A : - Before I don't know the idea how to solve the equation by figure. Aim is same but the process is different.

A : - Not only calculation, we can solve equation by figure too.

A : - I came to know that there are many way of thinking about solving mathematics, One of them is equation with figure.

Generally equation expressing with figure is called geometric representation. In the previous lectures we find that algebraic equation could represent with geometrical figure. In this world everything has own culture. As for example if we catch a bird and hanged it, after sometimes it must die. That means it is apt with its own culture. Similarly Arab has own culture. Technology means the development of science. We are studying mathematics so for the development of mathematics Cabri Geometry is the software whatever used to draw geometric figure.

Discussing the above questionnaire, conclusion is that students gathered knowledge on Modern Technology, Arab culture and Geometrical representation whatever my aim.

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5. CONCLUSION : -

The aim of this paper is to illustrate the significance of using modern technology, Arab culture, and Geometrical representation of algebraic equation, for this purposes it is used regional map of that country. Through three lecture there are using technology in the case of teaching mathematics history; specifically, mathematics as a human enterprise through giving students the chance to experience the hermeneutics in mathematics history. Overall we say that, teaching should not be separated from teaching with tradition. Both historical and new technologies are useful tools for knowing the imbedded socio-historical-cultural perspective in mathematics. Teaching history with both traditional and new technologies is effective for the change of belief in mathematics. We should expect near future mathematics teacher both historical and modern technology ability.

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Remarks : -

1. This project is under Professor MASAMI ISODA , University of Tsukuba, Japan.
2. This project is attached with a web page as follows http://www.mathedu-jp.org .

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