HIERONYMI CARDANI, PRÆSTANTISSIMI MATHEMATICI, PHILOSOPHI, AC MEDICI,

ARTIS MAGNÆ,

SIVE DE REGVLIS ALGEBRAICIS, Lib. unus. Qui & totius operis de Arithmetica, quod OPVS PERFECTVM inscripsit, est in ordine Decimus.

HAbes in hoc libro, studiose Lector, Regulas Algebraicas (Itali, de la Cosa uocant) nouis adinuentionibus, ac demonstrationibus ab Authore ita locupletatas, ut pro pauculis antea uulgò tritis, iam septuaginta euaserint. Neque solum, ubi unus numerus alteri, aut duo uni, uerum etiam, ubi duo duobus, aut tres uni equales fuerint, nodum explicant. Hunc aût librum ideo seorsim edere placuit, ut hoc abstrusissimo, & planè inexhausto totius Arithmeticæ thesauro in lucem eruto, & quasi in theatro quodam omnibus ad spectandum exposito, Lectores incitarêtur, ut reliquos Operis Perfecti libros, qui per Tomos edentur, tanto auidius amplectantur, ac minore fastidio perdiscant.

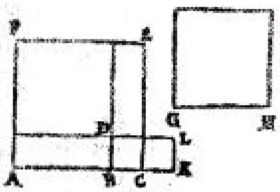
relinquitur prima 6 m: ℞ 30$\frac{6}{7}$, hæ autem quantitates proportionales ſunt, & quadratum ſecundæ eſt æquale duplo producti ſecundæ in primam, cum quadruplo primæ, ut proponebatur.

De cubo & rebus æqualibus numero. Cap. XI.

SCipio Ferreus Bononienſis iam annis ab hinc triginta ferme capitulum hoc inuenit, tradidit uero Anthonio Mariæ Florido Veneto, qui cũ in certamen cũ Nicolao Tartalea Brixellenſe aliquando ueniſſet, occaſionem dedit, ut Nicolaus inuenerit & ipſe, qui cum nobis rogantibus tradidiſſet, ſuppreſſa demonſtratione, freti hoc auxilio, demonſtrationem quæſiuimus, eamq; in modos, quod difficillimum fuit, redactam ſic ſubiecimus.

DEMONSTRATIO.

Sit igitur exempli cauſa cubus G H & ſexcuplum lateris G H æquale 20, & ponam duos cubos A E & C L, quorum differentia ſit 20, ita quod productum A C lateris, in C K latus, ſit 2, tertia ſcilicet numeri rerum pars, & abſcindam C B, æqualem C K, dico, quod ſi ita fuerit, lineam A B reſiduum, eſſe æqualem G H, & ideo rei æſtimationem, nam de G H iam ſupponebatur, quod ita eſſet, perficiam igitur per modum primi ſuppoſiti 6[i] capituli huius libri, corpora D A, D C, D E D F, ut per D C intelligamus cubum B C, per



D F cubum A B, per D A triplum C B in quadratum A B, per D E triplum A B in quadratũ B C. quia igitur ex A C in C K fit 2, ex A C in C K ter, fiet 6 numerus rerum, igitur ex A B in triplum A C in C K fiunt 6 res A B, ſeu ſexcuplum A B, quare triplum producti ex A B, B C, A C, eſt ſexcuplum A B, at uero differentia cubi A C, à cubo C K, & exiſtenti à cubo B C ei æq̃le ex ſuppoſito, eſt 20, & ex ſuppoſito primo 6[i] capituli, eſt aggregatum corporum D A, D E, D F, tria igitur hæc corpora ſunt 20, poſita uero B C m: cubus A B, æqualis eſt cubo A C, & triplo A C in quadratum C B, & cubo B C m: & triplo B C in quadratum A C m: per demonſtrata illic, differentia autem tripli B C in quadratum A C, à triplo A C in quadratum B C eſt productum A B, B C, A C, quare cum hoc, ut demonſtratum eſt, æquale ſit ſexcuplo A B, igitur addito ſexcuplo A B, ad id quod fit ex A C in quadratum B C ter, fiet triplum B C in quadratum A C, cum igitur B C ſit m: iam oſtenſum eſt, quod productum C B

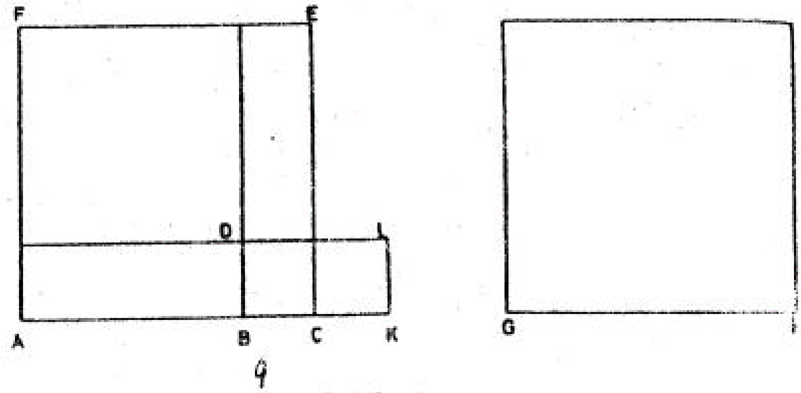
CONCERNING A CUBE AND UNKNOWNS EQUAL TO A NUMBER

Chapter XI

Scipio del Ferro of Bologna about thirty years ago invented [the method set forth in] this chapter, [and] communicated it to Antonio Maria Florido of Venice, who when he once engaged in a contest with Nicolo Tartaglia of Brescia announced that Nicolo also invented it: and he [Nicolo] communicated it to us when we asked for it, but suppressed the demonstration.[1] With this aid we sought the demonstration, and found it, though with great difficulty, in the manner which we set out in the following.

Demonstration. For example, let the cube of GH and six times the side GH be equal to 20.[2] I take [Fig. 1] two cubes AE and CL whose difference shall be 20, so that the product of the side AC by the side CK shall be 2, i.e., a third of the number of unknowns, and I lay off CB equal to CK; then I say that if it is done thus, the remaining line AB is equal to GH and therefore to the value of the unknown (for it was supposed of GH that it was so). Therefore I complete, after the manner of the first theorem of the 6th chapter of this book,[3] the solids DA, DC, DE, DF, so that we understand by DC the cube of BC, by DF the cube of AB, by DA three times CB times the square of AB, by DE three times AB times the square of BC. Since therefore from AC times CK the result is 2, from 3 times AC times CK will result 6, the number of unknowns, and therefore from AB times 3 AC times CK there results 6 unknowns AB, or 6 times AB, so that 3 times the product of AB, BC, and AC is 6 times AB. But the difference of the cube AC from the cube CK, and likewise from the cube BC, equal to it by hypothesis, is 20; and from the first theorem of the 6th chapter, this is the sum of the solids DA, DE, and DF, so that these three solids make 20. But taking BC minus, the cube of AB is equal to the cube of AC and 3 times AC into the square of CB and minus the cube of BC and minus 3 times BC into the square of AC. By the demonstration, the difference between 3 times CB times the

Fig. 1



in quadratum A C ter, eſt m: & reliquum quod ei æquatur eſt p: igitur triplum C B in q̃dratum A B, & triplum A C in q̃dratũ C B, & ſexcuplũ A B nihil faciunt. Tanta igitur eſt differentia, ex cõmuni animi ſententia, ipſius cubi A C, à cubo B C, quantum eſt quod cõflatur ex cubo A C, & triplo A C in quadratum C B, & triplo C B in quadratum A C m: & cubo B C m: & ſexcuplo A B, hoc igitur eſt 20, quia differentia cubi A C, à cubo C B, fuit 20, quare per ſecundum ſuppoſitum 6ⁱ capituli, poſita B C m: cubus A B æquabitur cubo A C, & triplo A C in quadratum B C, & cubo B C m: & triplo B C in quadratum A C m: cubus igitur A B, cum ſexcuplo A B, per communem animi ſententiam, cum æquetur cubo A C & triplo A C in quadratum C B, & triplo C B in quadratum A B m: & cubo C B m: & ſexcuplo A B, quæ iam æquatur 20, ut probatum eſt, æquabuntur etiam 20, cum igitur cubus A B & ſexcuplum A B æquentur 20, & cubus G H, cum ſexcuplo G H æquentur 20, erit ex communi animi ſententia, & ex dictis, in 35ᵃ pᵢ & 31ᵃ undecimi elementorum, G H æqualis A B, igitur G H eſt differentia A C & C B, ſunt autem A C & C B, vel A C & C K, numeri ſeu lineæ continentes ſuperficiem, æqualem tertiæ parti numeri rerum, quarum cubi differunt in numero æquationis, quare habebimus regulam.

Regula.

Deducito tertiam partem numeri rerum ad cubum, cui addes quadratum dimidij numeri æquationis, & totius accipe radicem, ſcilicet quadratam, quam ſeminabis, unicq; dimidium numeri quod iam in ſe duxeras, adijcies, ab altera dimidium idem minues, habebisq; Binomium cum ſua Apotome, inde detracta ℞ cubica Apotomæ ex ℞ cubica ſui Binomij, reſiduũ quod ex hoc relinquitur, eſt rei æſtimatio.

Exemplum. cubus & 6 poſitiones, æquantur 20, ducito 2, tertiam partem 6, ad cubum, fit 8, duc 10 dimidium numeri in ſe, fit 100, iunge 100 & 8, fit 108, accipe radicem quæ eſt ℞ 108, & eam geminabis, alteri addes 10, dimidium numeri, ab altero minues tantundem, habebis Binomiũ ℞ 108 p: 10, & Apotomen ℞ 108 m: 10, horum accipe ℞ᵃᵐ cubᵃᵐ & minue illam quę eſt Apotomæ, ab ea quæ eſt Binomij, habebis rei æſtimationem, ℞ v: cub: ℞ 108 p: 10 m: ℞ v: cubica ℞ 108 m: 10.

cubᵉ p: 6 rebᵉ æq̃lis 20
2 20
8 ——— 10
108
℞ 108 p: 10
℞ 108 m: 10
℞ v: cu. ℞ 108 p: 10
m: ℞ v: cu. ℞ 108 m: 10

Aliud, cubus p: 3 rebus æquetur 10, duc 1, tertiam partem 3, ad cubum, fit 1, duc 5, dimidium 10, ad quadratum, fit 25, iunge 25 & 1, square of AC, and 3 times AC times the square of BC, is [3 times] the product of AB, BC, and AC. Therefore since this, as has been shown, is equal to 6 times AB, adding 6 times AB to that which results from AC into 3 times the square of BC there results 3 times BC times the square of AC, since BC is minus. Now it has been shown that the product of CB[4] into 3 times the square of AC is minus; and the remainder which is equal to that is plus, hence 3 times CB into the square of AC and 3 times AC into the square of CB and 6 times AB make nothing. Accordingly, by common sense, the difference between the cubes AC and BC is as much as the totality of the cube of AC, and 3 times AC into the square of CB, and 3 times CB into the square of AC (minus), and the cube of BC (minus), and 6 times AB. This therefore is 20, since the difference of the cubes AC and CB was 20. Moreover, by the second theorem of the 6th chapter, putting BC minus, the cube of AB will be equal to the cube of AC and 3 times AC into the square of BC minus the cube of BC and minus 3 times BC into the square of AC. Therefore the cube of AB, with 6 times AB, by common sense, since it is equal to the cube of AC and 3 times AC into the square of CB, and minus 3 times CB into the square of AC, and minus the cube of CB and 6 times AB, which is now equal to 20, as has been shown, will also be equal to 20. Since therefore the cube of AB and 6 times AB will equal 20, and the cube of GH, together with 6 times GH, will equal 20, by common sense and from what has been said in the 35th and 31st of the 11th Book of the *Elements*, GH will be equal to AB, therefore GH is the difference of AC and CB. But AC and CB, or AC and CK, are numbers or lines containing an area equal to a third part of the number of unknowns whose cubes differ by the number in the equation, wherefore we have the

RULE[5]

Cube the third part of the number of unknowns, to which you add the square of half the number of the equation, and take the root of the whole, that is, the square root, which you will use, in the one case adding the half of the number which you just multiplied by itself, in the other case subtracting the same half, and you will have a binomial and apotome respectively; then subtract the cube root of the apotome from the cube root of the binomial, and the remainder from this is the value of the unknown. In the example, the cube and 6 unknowns equals 20; raise 2, the 3rd part of 6, to the cube, that makes 8; multiply 10, half the number, by itself, that makes 100; add 100 and 8, that makes 108; take the root, which is $\sqrt{108}$, and use this, in the first place adding 10, half the number, and in the second place subtracting the same amount, and you will have the binomial $\sqrt{108} + 10$, and the apotome $\sqrt{108} - 10$; take the cube root of these and subtract that of the apotome from that of the binomial, and you will have the value of the unknown $\sqrt[3]{\sqrt{108} + 10} - \sqrt[3]{\sqrt{108} - 10}$.

CONCERNING A CUBE AND UNKNOWNS EQUAL TO A NUMBER
第１１章

ボローニャのシピオーネ デル フェッロが30年程前、この章にある３次方程式の解法を発見し、ヴェニスのAntonio Maria Florido （フィオール）にそれを伝えている。彼は３次方程式の解法を発見したと公表したブレシアのニッコロ タルタリアと３次方程式について数学の試合をしている。私がタルタリアに頼んだところ、彼は解法を教えてくれたがその証明は教えてくれなかった。それを助けとして証明を探し求め、非常に困難だったがここで提示する方法を発見した。

demonstration（証明）

例としてGHの立方とGHの6倍が２０に等しいとしよう。ここで差が２０であり、各々の辺ACとCKの積が２、すなわちGHの個数の３分の１である２つの立方体AE、CLをとる。そしてCKに等しいCBをとる。もしこのようにできたならABが求めるGHと等しくなる。したがってこの本の第６章の最初の定理によって、立体DA、DC、DE、DFを完成することができ、よってDCをBCの立方、DFをABの立方、DAをCBの３倍とABの平方との積、DEをABの３倍とBCの平方との積であると理解できる。ACとCKの積は２であるから３倍のACとCKの積は６であり、これは未知数（GH）の個数である。したがってABと３倍のACとCKの積は６倍のABである。よって、AB、BC、ACの積の３倍は６倍のABである。しかしACの立方とCKまたはBCの立方との差は仮定より２０であり、また第６章の最初の定理から、これはDA、DE、DFの和である、すなわちこれらの３つの立体は２０をつくる。しかしBCを負にとると、ABの立方はACの立方と３倍のACとCBの平方との積との和から、BCの立方、３倍のBCとACの平方の積を引いたものである。証明によって、３倍のCBとACの平方との積と、３倍のACとBCの平方との積の差は、AB、BC、ACの積の３倍である。したがってこれは６倍のABであり、BCは負であるからACとBCの平方の３倍との積に６倍のABを加えると、３倍のBCとACの平方との積になる。よってCBとACの平方の３倍との積は負であることが示された。したがってこれに等しいものは正であり、それゆえ３倍のCBとACの平方との積、３倍のACとCBの平方との積、６倍のABの和は何もつくらない。それゆえACの立方とBCの立方との差は、ACの立方、３倍のACとCBの平方との積、３倍のCBとACの平方との積（負）、BCの立方（負）、６倍のABの総和である。ACの立方とCBの立方との差は２０であるから、これらは２０である。さらに第６章の２つ目の定理からBCを負とおくと、ABの立方は、ACの立方、３倍のACとBCの平方との積、BCの立方（負）、３倍の

BC と AC の平方との積（負）の総和である。したがって AB の立方と６倍の AB との和は、AC の立方、３倍の AC と CB の平方との積、３倍の CB と AC の平方との積（負)、CB の立方、６倍の AB の総和であるから２０である。したがって、AB の立方と６倍の AB の和は２０である。そして GH の立方と６倍の GH の和も２０であるから、原論１１巻の３１と３５でいわれているように GH と AB は等しいといえる。よって GH は AC と CB との差である。しかし AC と CB、または AC と CK は面積を含んだ数、線分であり、その面積は未知数の個数の３分の１に等しく、未知数の立方は方程式の中の(定)数とは異なる。それゆえに規則がある。

RULE

未知数の個数の３分の１の立方に定数の半分の平方を加え、全体の平方根をとり、ひとつの場合は定数の半分を加え、もうひとつの場合は定数の半分を引くと、binomial と apotome をそれぞれ得る。このとき、binomial の立方根から apotome の立方根を引き、残ったものが未知数の値である。例の中では立方と未知数の６倍が２０であり、６の３分の１である２をとる。その立方は８である。定数の半分である１０を平方すると 100 である。８と 100 を加え 108 となり、その平方根 $\sqrt{108}$ をとる。そしてこれに定数の半分である１０を加え、また１０を引くと $\sqrt{108}+10$ と $\sqrt{108}-10$ を得る。これらの立方根をとり、前者の立方根から後者の立方根を引いたものが求める未知数の値である。

すなわち $\sqrt[3]{\sqrt{108}+10}-\sqrt[3]{\sqrt{108}-10}$

カルダノの証明を追ってみよう！！

3年B組＿＿＿＿＿＿

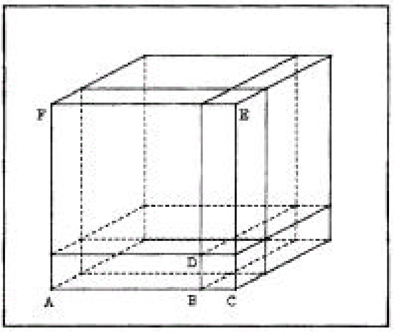
立方体ＡＥの体積・・・・u^3
立方体ＣＬの体積・・・・v^3 で $u^3-v^3=20$，$uv=(6/3)=2$ すると

おっきな立方体ＡＥは

立体ＤＡ・・・・・＿＿個でその体積は＿＿＿＿

立体ＤＣ・・・・・＿＿個でその体積は＿＿＿＿
立体ＤＥ・・・・・＿＿個でその体積は＿＿＿＿

立体ＤＦ・・・・・＿＿個でその体積は＿＿＿＿



でできてるよね！

つまり

$u^3=$＿＿＿＿＋＿＿＿＿＋＿＿＿＿＋＿＿＿＿
ここで立体ＤＡと立体ＤＥをくっつけて一つの
立体をつくるとその体積は＿＿＿＿となり

$u^3-v^3=$＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿
$=20$
いま
$x^3+6x=20$だから

$x=u-v$ となる.

（**参考文献** Struik, A Source Book in Math）